PART - A

(25 Marks)

1. Form the partial differential equation by eliminating arbitrary function f from
   \[ Z = f\left(\frac{xy}{z}\right) \]  
   3

2. Solve \((Z - px - qy) pq = p + q\).  
   2

3. Write the Euler’s formulae.  
   2

4. Find the Fourier coefficient \(a_0\) in the Fourier series expansion of
   \(f(x) = x \sin x\) in \([-\pi, \pi]\).  
   3

5. Solve \(3 u_x + 2 u_y = 0, u(x, 0) = 4 e^{-x}\).  
   3

6. Write one and two dimensional wave equations.  
   2

7. Find the Z transform of \(\langle \cos(n\theta) \rangle\).  
   3

8. Find \(Z^{-1}\left[\frac{3z}{(z - 3)^2}\right]\).  
   2
9. Find the Lagrange interpolating polynomial that fits the following data.

<table>
<thead>
<tr>
<th>x</th>
<th>2.5</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

10. Evaluate \( \int_{1}^{2} \cos x \, dx \) by Simpson's 3/8 rule with \( n = 2 \).

11. a) Solve \( (y + z)p + (z + x)q = x + y \)

b) Solve \( 2x z - px^2 - 2xyq + pq = 0 \) by Charpit's method.

12. a) Find the Fourier sine series of \( f(x) = 1 \) in \([0, 2]\)

b) Find the complex Fourier series of \( f(x) = e^{-x} \), \(-\pi < x < \pi\).

13. Solve \( \frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \), \( 0 < x < l, l > 0 \).

Subject to \( u(0, t) = u(l, t) = 0 \) and \( u(x, 0) = x ((l-x)) \).

14. a) If \( Z \{f_n\} = F(z) = \frac{3z^3 + 5z^2 - 7z + 1}{(z + 2)^2(z - 1)} \), find \( \lim_{n \to \infty} f_n \).

b) Solve \( y_{n+2} - 3 y_{n+1} + 2y_n = 0, y_0 = -1, y_1 = 2 \) using \( Z \) - transforms.
15. a) Perform the first three iterations of Gauss-Seidel iteration method to solve

\[
\begin{bmatrix}
4 & 1 & 2 \\
1 & 5 & 1 \\
2 & 1 & 4
\end{bmatrix}
\begin{bmatrix}
x \\ y \\ z
\end{bmatrix}
= 
\begin{bmatrix}
-1 \\
5 \\
3
\end{bmatrix}
\]

b) Using Runge-kutta 4th order method, find \( y(0.1) \) and \( y(0.2) \) for

\[ y' = xy + y^2, \quad y(0) = 1. \]

16. Solve \( r = 25t \) by Monge’s method.

17. a) Using convolution theorem, find \( Z^{-1}\left[ \frac{z^2}{(z-1)(z-2)} \right] \).

b) Find \( y(1.6) \) from

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>1.4</th>
<th>1.8</th>
<th>2.2</th>
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</thead>
<tbody>
<tr>
<td>y</td>
<td>3.49</td>
<td>4.82</td>
<td>5.96</td>
<td>6.5</td>
</tr>
</tbody>
</table>