PART – A (25 Marks)

1. Find a partial differential equation from \((x - a)^2 + (y - b)^2 + z^2=9\) by eliminating arbitrary constants \(a\) and \(b\). (3)

2. Solve \(p^3q^2x + p^2q^3y - zp^2q^2=1\). (2)

3. Find the Fourier coefficient 'a₁' of the Fourier series expansion of \(f(x)=x \sin x\) in \((0, 2\pi)\). (2)

4. Find the half-range sine series for \(e^x\) in \((0, 1)\). (3)

5. Solve \(\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}\) by the method of separation of variables. (3)

6. Show that \(u(x, y) = 6e^{-3x-2y}\) is a solution of \(\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} - u = 0\). (2)

7. Using Bisection method, find the first two approximations to the root of the equation \(x^3-4x-9=0\) which lies in \((2, 3)\). (3)

8. Construct Newton's divided difference table for:

<table>
<thead>
<tr>
<th>x</th>
<th>5</th>
<th>6</th>
<th>9</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>16</td>
</tr>
</tbody>
</table>

9. Determine the Z transform of \(\{e^{h2}\}\). (2)

10. If \(Z\{f_n\} = \frac{3z^2 - 4z + 7}{(z-1)^3}\), then find \(f_1\). (3)

PART – B (5x10=50 Marks)

11.(a) Solve \(x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)\). (5)

(b) Solve \(z = xy + pq\). (5)

12.(a) Explain \(f(x) = |\sin x|\), \(-\pi < x < \pi\) in a Fourier series. (5)

(b) Find the complex form of the Fourier series of the function

\[ f(x) = \begin{cases} 
0, & 0 < x < \ell \\
a, & \ell < x < 2\ell 
\end{cases} \]
13. A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is

\[ u(x, 0) = \begin{cases} 
  x, & 0 \leq x \leq 50 \\
  100 - x, & 50 \leq x \leq 100 
\end{cases} \]

Find the temperature \( u(x, t) \) at any time. \hspace{1cm} (10)

14. (a) Compute \( \frac{d^2 y}{dx^2} \) at \( x = 1 \) from the table

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>8</td>
<td>27</td>
<td>64</td>
<td>125</td>
<td>216</td>
</tr>
</tbody>
</table>

(b) Find the approximate value of \( y(1.1) \) for \( \frac{dy}{dx} = 3x + y^2 \), \( y(1) = 1.2 \) by Runge-Kutta method of fourth order. \hspace{1cm} (5)

15. (a) Using convolution theorem, find the inverse z transform of \( \frac{3}{z(z-1)} \). \hspace{1cm} (5)

(b) Solve \( y_{n+2} - 3y_{n+1} + 2y_n = 0 \), \( y_0 = -1 \), \( y_1 = 2 \) using Z transform. \hspace{1cm} (5)

16. Solve \( r - t \cos^2 x + p \tan x = 0 \) using Monge's method. \hspace{1cm} (10)

17. (a) Perform the first two iterations of Gauss-Seidel iteration method to solve \( 4x - y + z = 4 \), \\
\( -x + 4y - z = 2 \) and \( x - y + 4z = 4 \). \hspace{1cm} (5)

(b) State and prove initial value theorem. \hspace{1cm} (5)